USN | | | | | | | | | | | |

BMATM201

Second Semester B.E./B.Tech. Degree Supplementary Examination, June/July 2024
Mathematics – II for Mechanical Engineering Stream

Time: 3 hrs.   Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$	7	L3	CO1
	b.	Evaluate $\int_{0}^{3}\int_{0}^{\sqrt{4-y}}(x+y)\,dx\,dy$ by changing the order of integration.	7	L3	CO1
	c.	Show that $\int_{0}^{\infty}\sqrt{y}\,e^{-y^2}\,dy \times \int_{0}^{\infty}\dfrac{e-y^2}{\sqrt{y}}\,dy = \dfrac{\pi}{2\sqrt{2}}$	6	L2	CO1
		OR			
Q.2	a.	Show that $\beta(m,n) = \dfrac{\Gamma_{(m)}\cdot\Gamma_{(n)}}{\Gamma_{(m+n)}}$	7	L2	CO1
	b.	Evaluate $\int_{0}^{a}\int_{0}^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\,dx\,dy$ by changing into polars.	7	L3	CO1
	c.	Write a modern mathematical tool program to find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$ and $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$.	6	L3	CO5
		Module – 2			
Q.3	a.	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.	7	L2	CO2
	b.	If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y)\hat{j} - 2x^3z^2\,k$, find div $\vec{F}$ and curl $\vec{F}$.	7	L2	CO2
	c.	Find the constants a, b and c such that the vector $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational.	6	L2	CO2
		OR			
Q.4	a.	Using Grun's theorem, evaluate $\int_{C}(2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$, where C is the boundary of the region in the xy-plane enclosed by the x-axis and the upper-half of the circle $x^2 + y^2 = a^2$.	7	L3	CO2

	b.	If $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the rectangular parallelopiped bounded by $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 1$, $z = 3$. Find the Flux across S.	7	L3	CO2
	c.	Write the modern mathematical tool program to find the divergence of the vector field $\vec{F} = (3x^2 - 3yz)i + (3y^2 - 3xz)j + (3z^2 - 3xy)k$	6	L3	CO5

Module – 3

Q.5	a.	Form the partial differential equation by eliminating the arbitrary function ϕ from $\ell x + my + nz = \phi\,(x^2 + y^2 + z^2)$.	7	L2	CO3
	b.	Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{x}{y}$, subject to the conditions that $\dfrac{\partial z}{\partial x} = \log_e x$ when $y = 1$ and $z = 0$ when $x = 1$.	7	L3	CO3
	c.	Derive the dimensional heat equation in the standard form $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$.	6	L2	CO3

OR

Q.6	a.	Form the partial differential equation by eliminating the arbitrary constants a and b from $2z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$.	7	L2	CO3
	b.	Solve $\dfrac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$, $\dfrac{\partial z}{\partial x} = a \sin y$ and $\dfrac{\partial z}{\partial y} = 0$.	7	L3	CO3
	c.	Solve $(y - z)\,p + (z - x)\,q = (x - y)$.	6	L2	CO3

Module – 4

Q.7	a.	Find an approximate value of the root of the equation $\cos x = 3x - 1$ that lies between 0.5 and 1 correct to three decimal places using Regula false method.	7	L3	CO4
	b.	The following table gives the distances (in miles) of the visible horizon for the given heights (in feet) above the earth's surface.	7	L3	CO4

x	200	250	300	350	400
F(x)	15.04	16.81	18.42	19.9	21.27

Find y for $x = 218$.

	c.	Evaluate $\displaystyle\int_0^{\pi} e^{\sin \theta}\, d\theta$ by using Simpson's $\left(\dfrac{1}{3}\right)^{rd}$ rule by taking 7 ordinates.	6	L3	CO4

OR

Q.8	a.	Using the Newton-Raphson method, find the real root of the equation $e^x \sin x = 1$. (Here x is in radians).	7	L3	CO4

	b.	Using Newton's divided difference formula, evaluate F(9) from the following table.	7	L2	CO4

x	5	7	11	13	17
F(x)	150	392	1452	2366	5202

	c.	If y(0) = -12, y(1) = 0, y(3) = 6 and y(4) = 12, find the value of y at x = 2 using Lagranges method.	6	L3	CO4

Module – 5

Q.9	a.	Find by Taylor's series method the value of y at x = 0.1 to 4 decimal places from $\frac{dy}{dx} = x - y^2$, y(0) = 1.	7	L2	CO4
	b.	Using the Runge-Kutta method of fourth order, find y(1.1) given that $\frac{dy}{dx} = xy^{1/3}$ taking h = 0.1.	7	L3	CO4
	c.	Given that $\frac{dy}{dx} = 2e^x - y$ and the data y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090. Compute y at x = 0.4 by applying Milne's method.	6	L3	CO4

OR

Q.10	a.	Using the modified Euler's method, find y(20.2) given that $\frac{dy}{dx} = \log_{10}(x/y)$ with y(20) = 5 taking h = 0.2.	7	L3	CO4
	b.	Apply the Runge-Kutta method of fourth order, to find an approximate value of y at x = 0.1, given that $\frac{dy}{dx} = 3e^x + 2y$ with y(0) = 0 and h = 0.1	7	L3	CO4
	c.	Using modern mathematical tools write a program to find y at x = 1.4, given $\frac{dy}{dx} = x^2 + \frac{y}{2}$, y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. Use corrector formula thrice using Milne's method.	6	L3	CO5

* * * * *